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## CY2550: Foundations of Cybersecurity Section 03

Crypto Module: Public-key cryptography, Diffie Hellman, RSA, hash functions, integrity and authenticated encryption, digital signatures, authenticity of public keys, TLS

## Outline

- Public-key cryptography
- Diffie Hellman Key Exchange
- RSA Encryption
- Cryptographic hash functions
- Integrity and authenticated encryption
- Digital signatures
- Authenticity of public keys
- TLS



#### Public Key Cryptography

## Limitation of symmetric key crypto

- How do you securely exchange keys with someone?
- Easy(ish) to do if you can meet them in person
- However, the Internet is untrusted
  - You can't exchange shared secrets over an untrusted medium







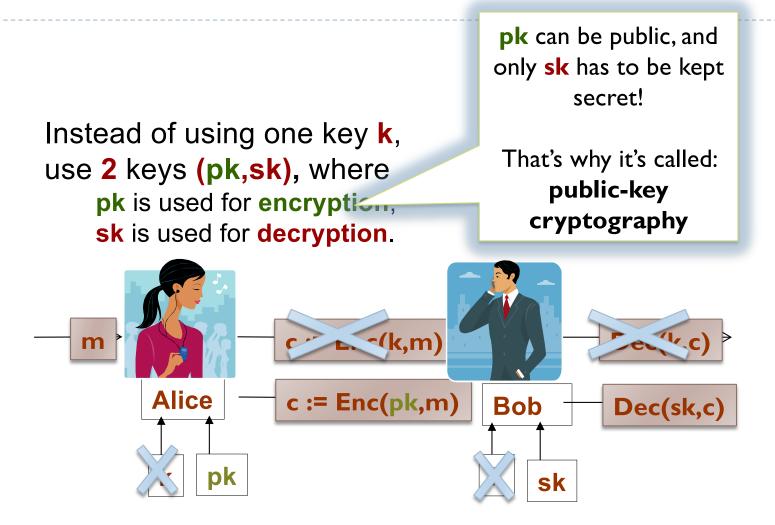
Eve

Bob

## Public Key Cryptography

- Public key cryptography, a.k.a. asymmetric cryptography
  - Each principal has two keys: private (secret) and public
  - A message encrypted with one key must be decrypted by the other
  - Thus, the public key can be sent in-the-clear over the Internet
- Security is based on Very Hard Math Problems
  - Fast to verify a given solution for a given instance
  - Hard to finds solutions for a given instance in polynomial time
- Many different algorithms that offer different security properties
  - Diffie-Hellman, RSA, Goldwasser-Micali, ElGamal
- Forms the basis for most modern secure communication protocols
  - ▶ IPsec, SSL, TLS, S/MIME, PGP/GPG, etc.

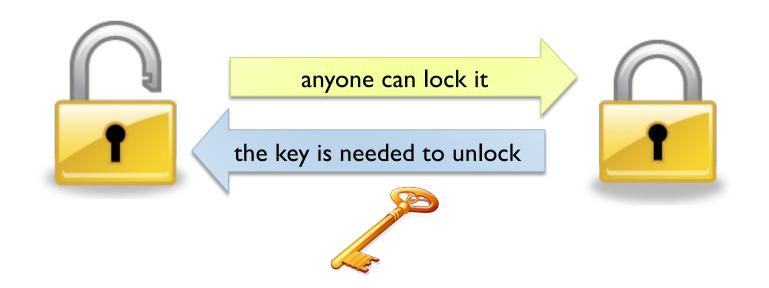
## Public Key Encryption







#### Examples padlocks:





## Use of Public-Key Cryptography

#### Public-Key Encryption

Examples: RSA, ElGamal

#### Digital Signatures:

- Authenticate messages
- Examples: RSA, DSA

#### Key Exchange

- Protocols to establish a secret key between two parties
- Examples: Diffie-Hellman

#### Intuition for all these

- Computation in one direction is "easy", but "hard" in the reverse
- Hardness assumptions imply that adversary cannot reverse computation



## A little bit of history

• **Diffie and Hellman** were the first to publish a paper containing the idea of the public-key cryptography:

W.Diffie and M.E.Hellman, **New directions in cryptography** IEEE Trans. Inform. Theory, IT-22, 6, **1976**, pp.644-654.

- A similar idea was described by **Ralph Merkle**:
  - in 1974 he described it in a project proposal for a Computer Security course at UC Berkeley (it was rejected)
  - in **1975** he submitted it to the CACM journal (it was rejected)

(see <a href="http://www.merkle.com/1974/">http://www.merkle.com/1974/</a> )

- I 977: R. Rivest, A. Shamir and L.Adelman published the first construction of publickey encryption (RSA)
- It 1997 the GCHQ (the British equivalent of the NSA) revealed that they knew it already in 1973.



#### Diffie Hellman Key Exchange

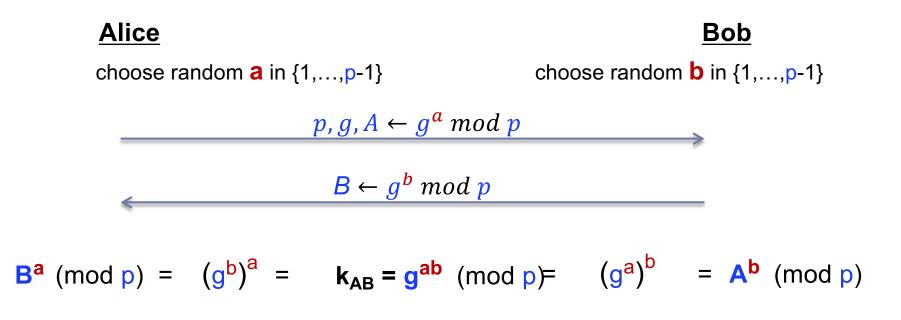
## Diffie-Hellman Key Exchange (Turing Award 2015)

Goal

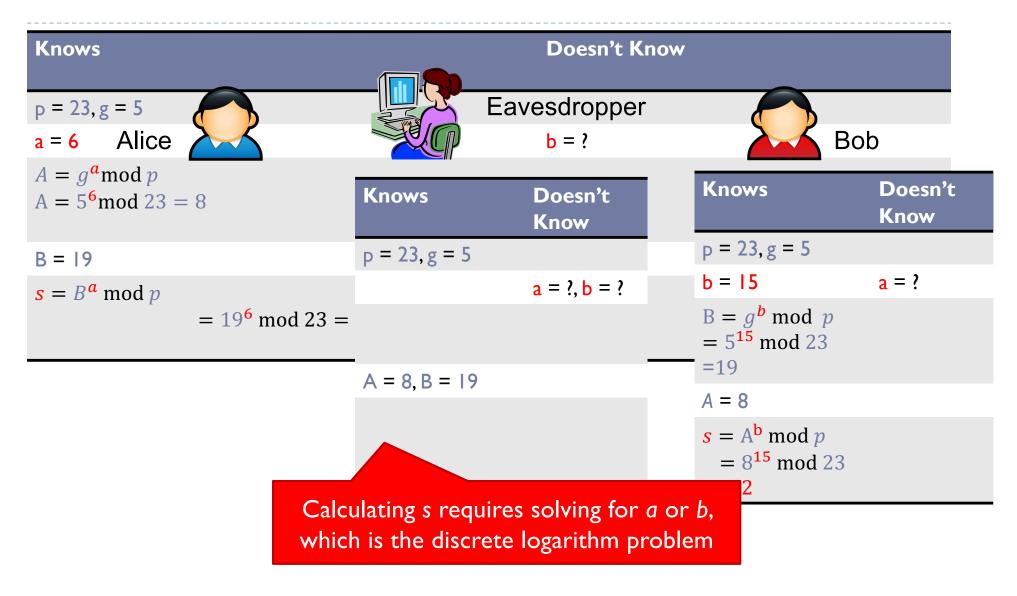
- Share a secret key over a public channel in presence of eavesdropping adversary
- Really should be called Diffie-Hellman-Merkle
  - Ralph Merkle developed the mathematical theories
  - Whitfield Diffie and Martin Hellman developed the protocol
- Security is based on the discrete logarithm problem
  - Compute k such that b<sup>k</sup> = g mod p, where b, g, and k are all integers and p is a large prime
  - Possible that no solution exists given arbitrary b and g
  - Best known algorithms are exponential time

## The Diffie-Hellman protocol

Fix a large prime p (e.g. 600 digits) Fix an integer g in {1, ..., p}

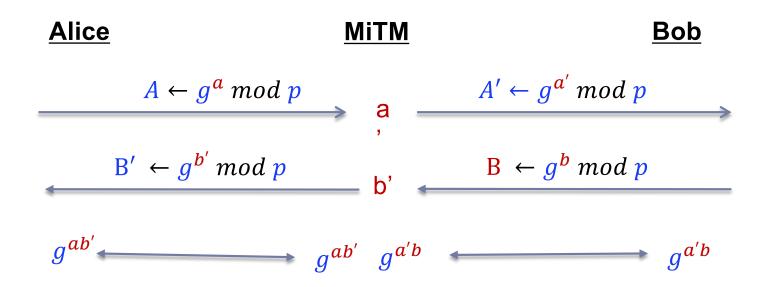


## Diffie-Hellman Example



## Man-in-the-middle attack

As described, the protocol is insecure against **active** attacks



Attacker relays traffic from Alice to Bob and reads it in clear

We will see later in the class how to fix this

RSA

- Encryption algorithm: Enc(pk, m); decryption Dec(sk, c)
- RSA algorithm invented by Rivest, Shamir, and Adleman in 1978
  - Equivalent system invented by Clifford Cox in 1973, but GCHQ classified it
- RSA is the dominant public key cryptosystem today
  - Algorithm was commercialized by RSA Security
  - RSA Security created a certificate authority that eventually became Verisign

## RSA Algorithm

- Security is based on the difficulty of factoring the product of primes
  - Alice chooses two secret primes p and q, n = pq,  $\phi(n) = (p 1)(q 1)$
  - Choose e such that  $I < e < \phi(n)$ , and  $gcd(e, \phi(n)) = I$
  - <n, e> is Alice's public key
  - Private key  $d = e^{-1} \mod \phi(n); d \cdot e = 1 \mod \phi(n)$
- Encryption and decryption
  - Given a message M, 0 < M < n
  - Compute ciphertext  $C = M^e \mod n$
  - To decipher, compute  $C^d \mod n = (M^e \mod n)^d \mod n = M^{e^d} \mod n = M$
- Use Euler's theorem:  $x^{\phi(n)} = 1 \mod n$

## RSA Example

$$p = 11, q = 7, n = pq = 77, \phi(n) = 60$$
  
 $e = 37, d = 13 (ed = 481, ed \mod 60 = 1)$   
If  $M = 15$  then  $C = M^e \mod n = 15^{37} \mod 77 = 71$   
 $C^d \mod n = 71^{13} \mod 77 = 15 = M$ 

# IND-CPA security for Public-Key Encryption

- In public-key encryption, everyone knows the public key
- That means everyone (including the adversary) can encrypt any message
- IND-CPA and IND-EAV are equivalent notions of security!
- Another reason why we demand IND-CPA at a minimum for symmetric-key encryption

## Plain RSA Encryption

Plain (textbook) RSA encryption:

- ▶ public key: < n, e > Encrypt:  $c \leftarrow M^e \mod n$
- ▶ secret key:  $< p, q, d > Decrypt: c^d \rightarrow M \mod n$

#### Insecure cryptosystem !! !

- Is not IND-CPA secure and many attacks exist
- Deterministic (public key) encryption is never IND-CPA secure

## Attacks Against RSA

- The length of n=pq reflects the strength
  - 700-bit *n* factored in 2007
  - 768 bit factored in 2009
- I024 bit for minimal level of security today
  - Likely to be breakable in near future
  - Recommended use of 2048 or 4096 bits
- RSA encryption/decryption speed is quadratic in key length

## **Computationally Hard Problems**

#### RSA problem:

 Given public RSA key, decrypt m<sup>e</sup> mod n for a random message m

#### RSA assumption:

Solving the RSA problem is difficult

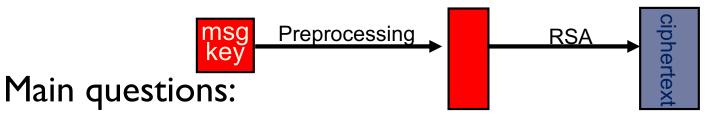
#### Factoring assumption

- ▶ If *n*=*pq* with *p* and *q* primes, cannot factor for large *n*
- If factoring can be done in polynomial time, then RSA problem can be solved in polynomial time

## RSA encryption in practice

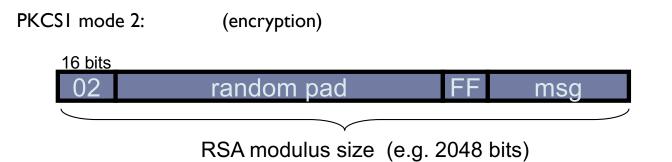
Never use plain RSA.

RSA in practice



- How should the preprocessing be done?
- Can we argue about security of resulting system?
- How can we randomize it to be IND-CPA secure?

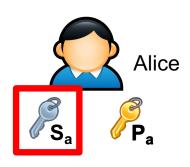
## PKCS1 v1.5



- Add random pad before the message
- Resulting value is RSA encrypted
- Widely deployed, e.g. in HTTPS, but it is not IND-CPA secure!
- There are newer versions that are secure (e.g., OAEP)



## Public Key Crypto Example

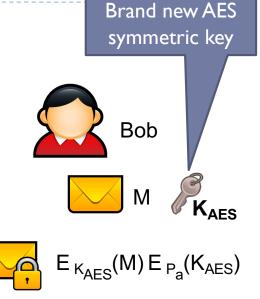






Eavesdropper





- Why bother with the symmetric key?
  - Why not just encrypt M with  $P_a$ ?
- Performance

 $E_{K_{AES}}(M) E_{P_a}(K_{AES})$ 

- Asymmetric crypto is slow, symmetric is fast
- Use asymmetric for K (which is small)
- Use symmetric for M (which is large)

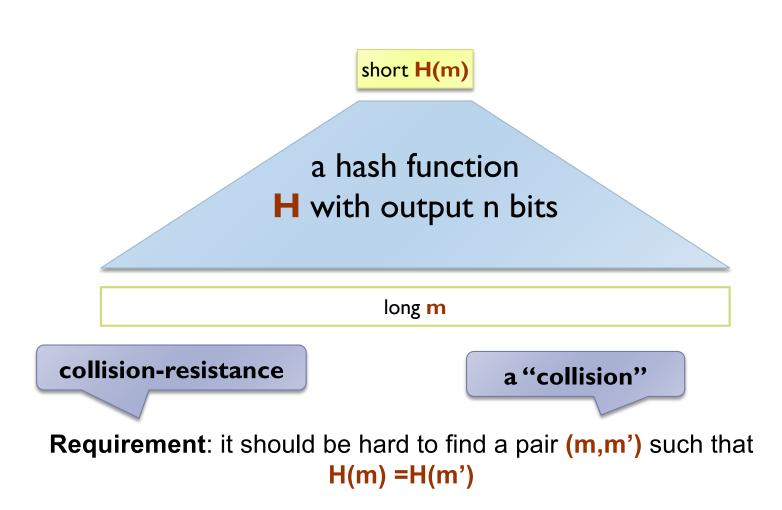
Key sharing can be done with a Key Exchange protocol (e.g., Diffie-Hellman)

#### Cryptographic hash functions

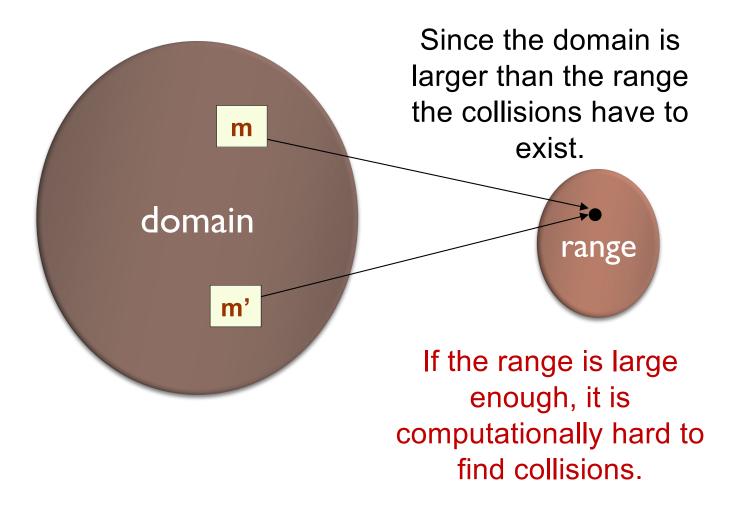
## **Cryptographic Hash Functions**

- Cryptographic hash function transform input data into scrambled output data
  - Arbitrary length input  $\rightarrow$  fixed length output
  - Deterministic: H(A) is always the same
  - High entropy:
    - md5('security') = e91e6348157868de9dd8b25c81aebfb9
    - md5('security1') = 8632c375e9eba096df51844a5a43ae93
    - md5('Security') = 2fae32629d4ef4fc6341f1751b405e45
  - Collision resistant
    - Locating A' such that H(A) = H(A') takes a long time
    - Example: 2<sup>21</sup> tries for md5

## Collision-resistant hash functions



## Collisions always exist



## Examples

Are these hash functions collision resistant?

- ▶ H:{0, I}<sup>2n</sup> -> {0, I}<sup>n</sup>
  - ► H(x||y) = x XOR y
- ▶ H:{0, I}<sup>2n</sup> -> {0, I}<sup>n</sup>
  - Let p be an n-bit prime
  - H(x||y) = x + y mod p
- ▶ H: N -> {0, I}<sup>n</sup>
  - Let p be an n-bit prime
  - $H(x) = ax + b \mod p, p \text{ prime}$

## History of hash functions

## **H** is a **collision-resistant hash function** if it is *"practically impossible to find collisions in H".*

- **1991**: MD5
- **1995**: SHA1
- 2001: SHA2 -- SHA-256 and SHA-512
- 2004: Team of Chinese researchers found collisions in MD5
- 2007: NIST competition for new SHA3 standard
- 2012: Winner of SHA3 is Keccak

## Well known hash functions

### MD5

- Outputs 128 bits
- Collision resistance totally broken in 2004
- SHAI
  - Outputs 160 bits
  - Partially broken: method exists to find collisions in 2<sup>80</sup> tries
  - Deprecated

## SHA2 family (SHA-224, SHA-256, SHA-384, SHA-512)

- SHA-224 matches the 112 bit key length of 3DES
- SHA-256, SHA-384, SHA-512 match the key lengths of AES (128, 192, 256 bits)
- Considered safe

## SHA3

- 2007: NIST opens competition for new hash functions
- > 2008: Submission deadline, 64 entries, 51 make the cut
- 2009: 14 candidates move to round 2
- 2010: 5 candidates move to round 3
- 2011: final round of public comments
- 2012: NIST selects keccak (pronounced "catch-ack") as SHA3
  - Created by Guido Bertoni, Joan Daemen, Gilles Van Assche, Michaël Peeters

## Birthday paradox

If we choose q elements y<sub>1</sub>, ... y<sub>q</sub> at random from {1,...,N}, what is the probability that there exists i and j such that y<sub>i</sub> = y<sub>j</sub> ?



- What is the probability that two people have the same birthday?
- When is this probability higher than 0.5?

## Collision probability

0.9 0.8 0.7 Collision probability 0.6 0.5 0.4 0.3 0.2 0.1 0 500 1000 2500 0 1500 2000 3000 3500 4000 4500 5000 # samples n

- If  $q = \Theta(\sqrt{N})$  items, then probability of collision is approx.  $\frac{1}{2}$
- Birthday paradox

Hash functions

$$N = 2^{256}, q = 2^{128}$$

 Implies n/2 level of security for n-bit hash function in best case

N=10<sup>6</sup>

## Security of encryption vs hash functions

#### • Encryption:

- Size of key determines resilience to brute-force attacks
- A key size of n bits, means 2<sup>n</sup> work for the attacker to find the key

#### Hash functions:

- Size of the output of the hash function determines security to finding collisions
- Birthday paradox tells us that for a hash function with on output of n bits, the work that an attacker needs to do to find a collision is 2<sup>n/2</sup>

#### For comparable security a block cipher of 128 bits must be paired with a hash function of 256 bits!

### Applications of hash functions

- Password authentication, store a hash of the password
  - We will see more about this in about 2 weeks when we will talk about authentication and passwords

- Integrity
  - Communication
  - Storage

#### Integrity and authenticated encryption

# Integrity

#### Active adversaries

- Can modify messages/ciphertexts in transit
- Encryption alone (even IND-CPA secure) does not guarantee integrity!

#### Protect message integrity

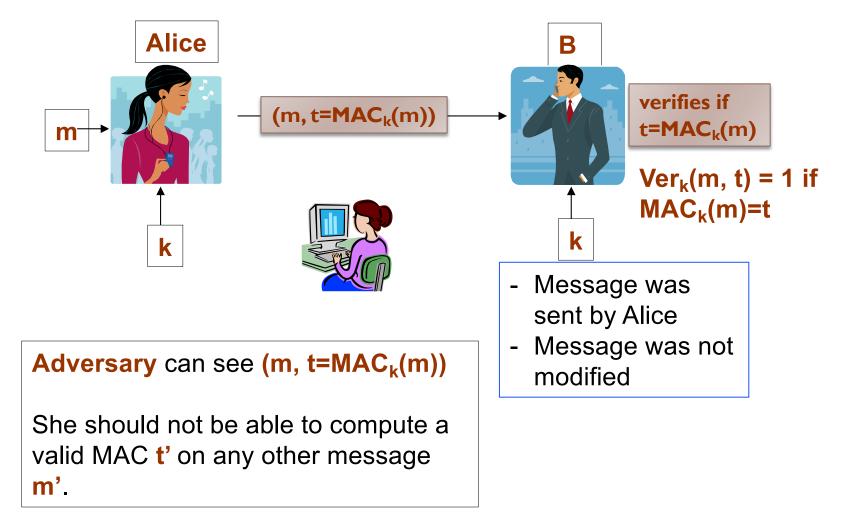
- Message received by Bob is the original one sent by Alice
- Message was not modified by adversary

#### Scenarios

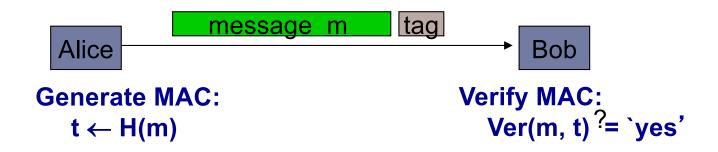
- Secure communication on network
- Protect files stored on disk
- Can be achieved in symmetric-key or public-key settings

Message authentication

MAC = message authentication code



### Integrity requires a secret key



- Attacker can easily modify message m and re-compute the hash.
- Hash designed to detect **random**, not malicious errors.

### Message authentication security

#### Properties

- Correctness: If  $t = MAC_k(m)$ , then  $Ver_k(m, t) = 1$
- ► *MAC* is a deterministic function
- The output of *MAC* is fixed size (n bits), independent of the length of the input message

### Security (unforgeability)

- If an attacker has many pairs of messages and integrity tags, he cannot compute a new tag on a message
- ▶ If A is given  $(m_1, t_1), ..., (m_q, t_q)$  then A cannot output (m', t')such that:  $Ver_k(m', t') = 1$  and  $m' \notin \{m_1, ..., m_q\}$

### Example: protecting system files

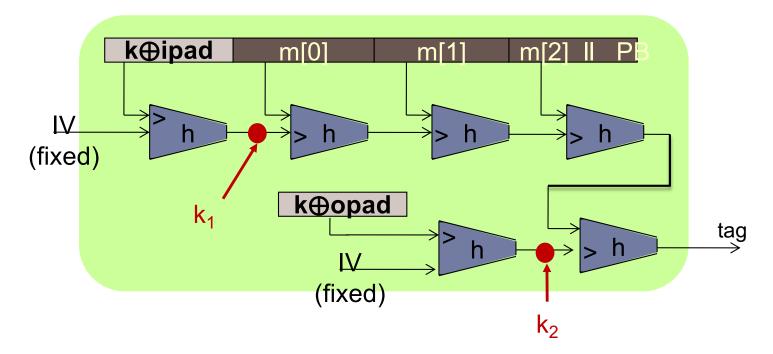
Suppose at install time the system computes:



k derived from user's password (e.g., using a hash)

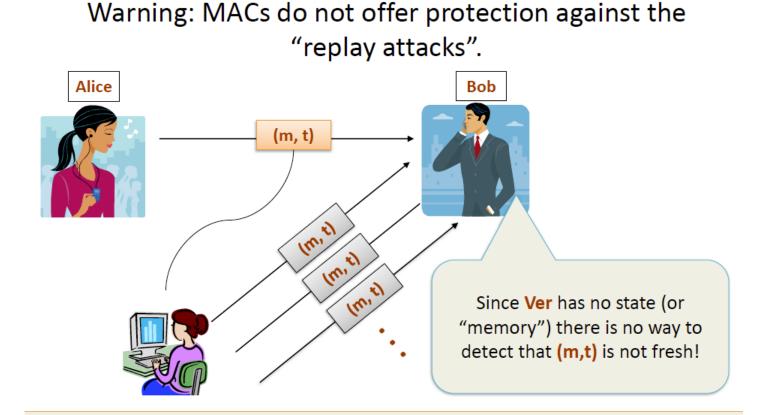
Later malware infects system and modifies system files
User reboots into clean OS and supplies his password
▶ Then: secure MAC ⇒ all modified files will be detected

### HMAC: Design a MAC from a hash function



- Uses Merkle-Damgaard construction (chaining a collision-resistant hash function)
- Output: last hashed block (no need to recover the message)
- Most widely used MAC on the Internet

### Replay attacks

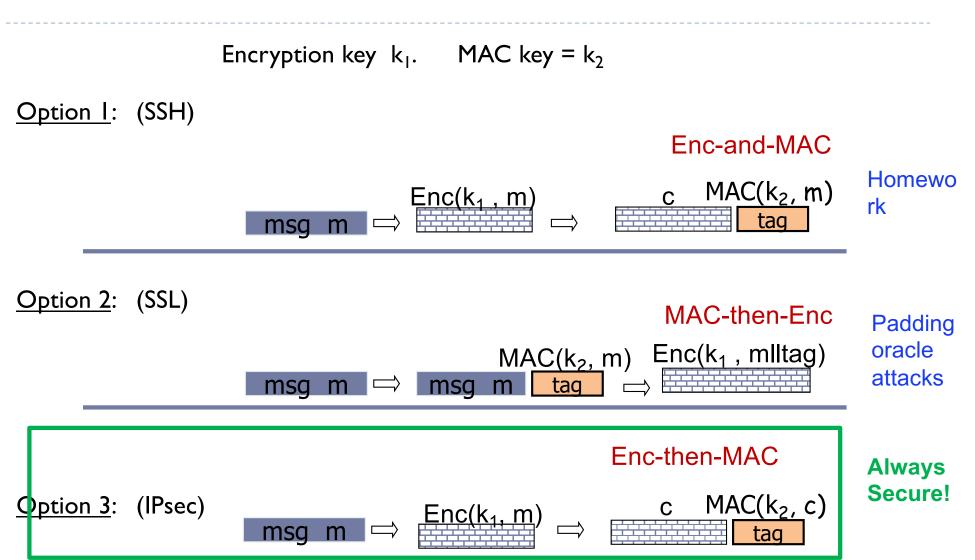


This problem has to be solved by the higher-level application (methods: time-stamping, sequence numbers...).

### Authenticated encryption

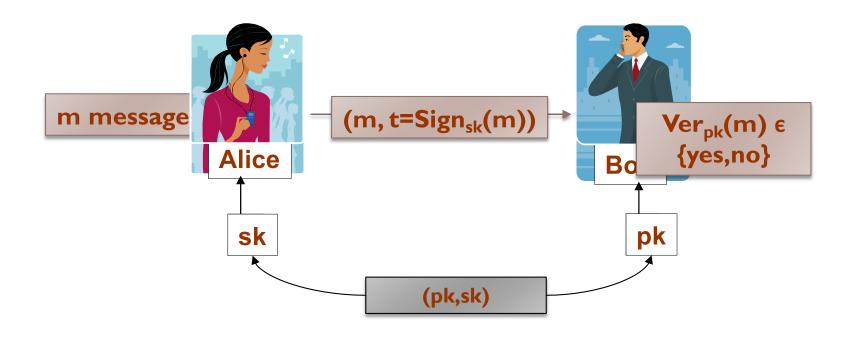
- Combines confidentiality and integrity
- Security properties
  - Confidentiality: ciphertext does not leak any information about the plaintext
  - Integrity: attacker cannot create new ciphertexts that decrypt properly
- Decryption returns either
  - Valid messages
  - Or invalid symbol (when ciphertext is not valid)

# Authenticated Encryption: combining MAC and ENC



#### Digital signatures

### Digital signature schemes



Public key equivalent of MAC

# Digital signatures and hash functions



- What can you infer about a signed message?
  - The holder of S<sub>a</sub> must have produced the signature
  - The message was not modified, otherwise the hash would not match
  - Assuming hash is collision resistant

Advantages of signature schemes

Digital signatures are equivalent of MACs in public-key world

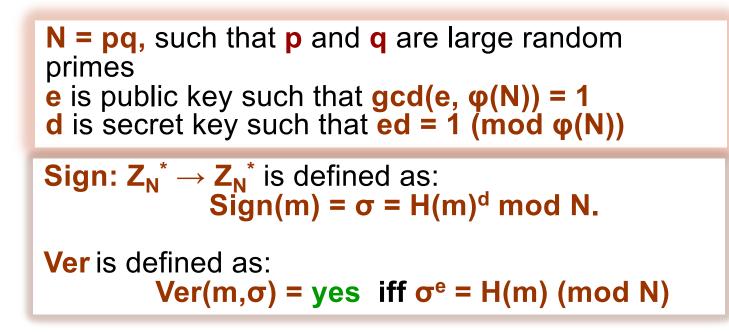
Provide message integrity

Additional properties (compared to MACs):

- I. **Publicly verifiable:** anyone with PK can verify (not for MAC)
- 2. **Transferable:** can be transferred to another user and verified
- 3. Provide **non-repudiation:** cannot deny that you signed a message

### RSA signature

Before computing the RSA function – apply hash function H.



Hash-and-sign paradigm

### Encryption vs. signatures

#### **Encryption**

#### What does encryption give you?

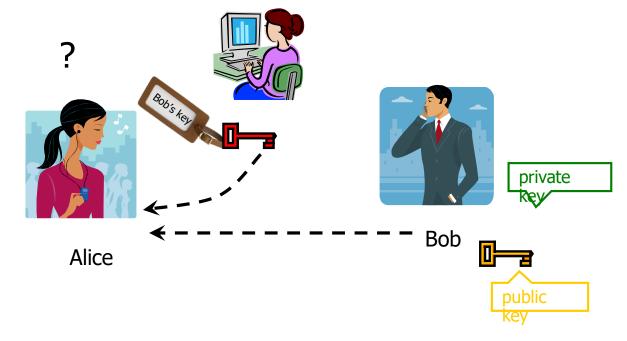
- Confidentiality only the holder of the private key can read the message
- What does authenticated encryption give you in addition?
  - Integrity if the ciphertext is modified, it will no longer decrypt properly
- What does encryption not give you?
  - Authentication you have no idea who used your public key to encrypt the message

#### **Digital Signatures**

- What do signatures give you?
  - (Weak) Authentication only the holder of the private key could have signed the message
  - Integrity if the message is modified, the signature will be invalid
- What do signatures not give you?
  - Confidentiality the message is not encrypted, it's public

#### Authenticity of public keys.

### Authenticity of Public Keys



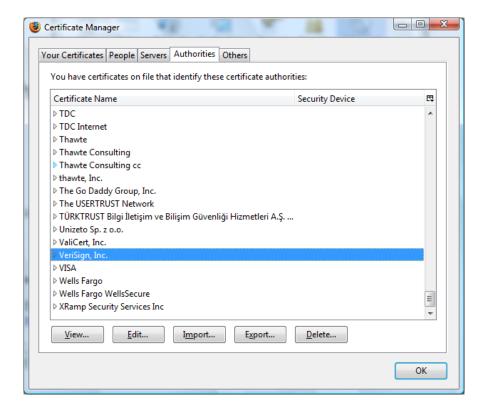
<u>Problem</u>: How does Alice know that the public key she received is really Bob's public key?

### PKI: Public Key Infrastructure

### Public announcement or public directory

- Risks: forgery and tampering
- Public-key certificate
  - Signed statement specifying the key and identity
    - Sig<sub>Charlie</sub>("Bob", PK<sub>Bob</sub>)
  - Could Bob sign his own certificate?
  - Web of trust (PGP): users signing each other's keys
- Common approach: certificate authority (CA)
  - An agency responsible for certifying public keys
  - It generates certificates for domain names (example.com) on the web

### **Trusted Certificate Authorities**



## Warning

				hop.co.uk	MyYaCdn	¥ yahooSports	3	
×								
Your connection is not private								
	-	s might be trying to steal your information from www.pcwebshop.co.uk (for , passwords, messages, or credit cards). NET::ERR_CERT_COMMON_NAME_INVALI						
Automatically report details of possible security incidents to Google. Privacy policy								
HIDE AD	VANCED					Bac	k to safety	
This ser	ver could	not prov	e that it	is www.pcv	vebshop.co.u	uk; its security cer	tificate is	
from *.secure-secure.co.uk. This may be caused by a misconfiguration or an attacker intercepting your connection. <u>Learn more</u> .								

Proceed to www.pcwebshop.co.uk (unsafe)

Figure 8.3: Self-signed site certificate—browser warning (Chrome 56.0.2924.87). The "Back to safety" tab discourages clicking-through to the site despite the browser being unable to verify the certificate chain (this happens when one or more certificates are signed by CAs unrecognized by the browser, i.e., not verifiable using the browser's trust anchors).

## CA Hierarchy or PKI

- Browsers, operating systems, etc. have trusted root certificate authorities
  - Firefox 3 includes certificates of 135 trusted root CAs
- A Root CA signs certificates for intermediate CAs, they sign certificates for lower-level CAs, etc.
  - Certificate "chain of trust"
    - Sig<sub>Verisign</sub>("neu.edu", PK<sub>NEU</sub>), Sig<sub>NEU</sub>("ccs.neu.edu", PK<sub>CCS</sub>)

### CA responsibilities

- Verify that someone buying a cert for a domain (e.g., example.com) actually controls the domain
- Verify that buyer knows the secret key associated with the public key
- Protect its own secret key

### Comodo

# Independent Iranian hacker claims responsibility for Comodo hack

Posts claiming to be from an Iranian hacker responsible for the Comodo hack ...

by Peter Bright - Mar 28 2011, 11:15am EDT

Hello
 I'm writing this to the world, so you'll know more about me..
 I'm writing this to the world, so you'll know more about me..
 At first I want to give some points, so you'll be sure I'm the hacker:
 At first I want to give some points, so you'll be sure I'm the hacker:
 I hacked Comodo from InstantSSL.it, their CEO's e-mail address mfpenco@mfpenco.com
 Their Comodo username/password was: user: gtadmin password: [trimmed]
 Their DB name was: globaltrust and instantsslcms

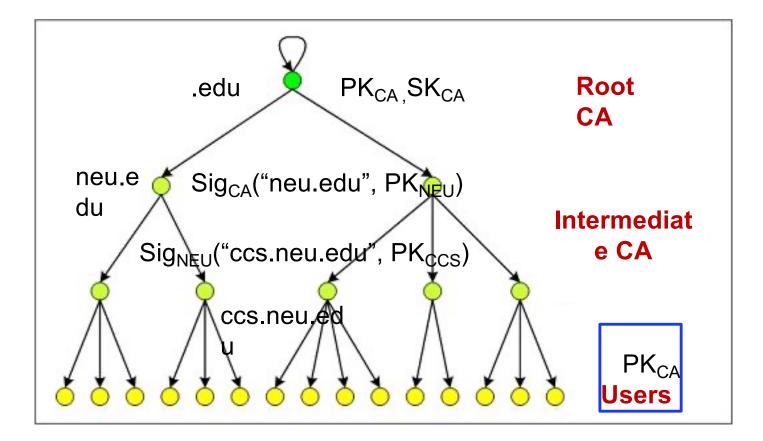
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The alleged hacker's claim of responsibility on pastebin.com

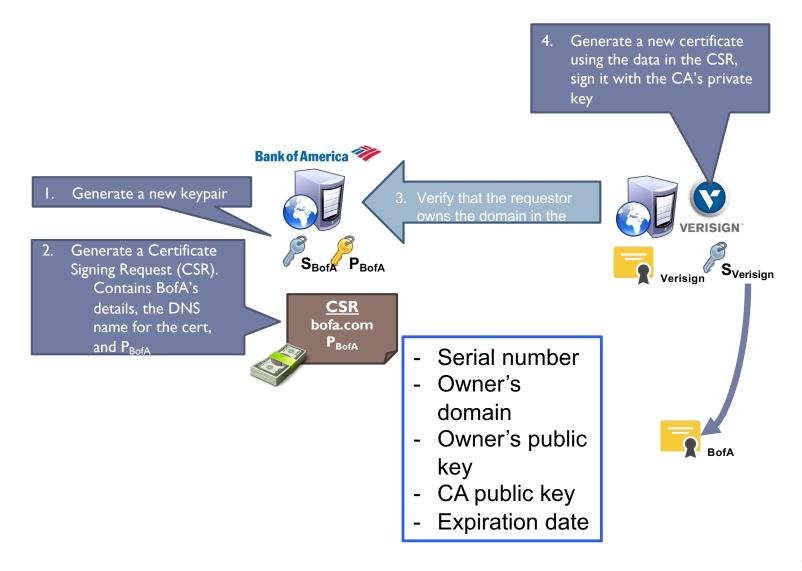
The hack that resulted in Comodo creating certificates for popular e-mail providers including Google Gmail, Yahoo Mail, and Microsoft Hotmail has been claimed as the work of an independent Iranian patriot. A post made to data sharing site pastebin.com by a person going by the handle "comodohacker" claimed responsibility for the hack and described details of the attack. A second post provided source code apparently reverse-engineered as one of the parts of the attack.

# What if CA secret key is compromised?

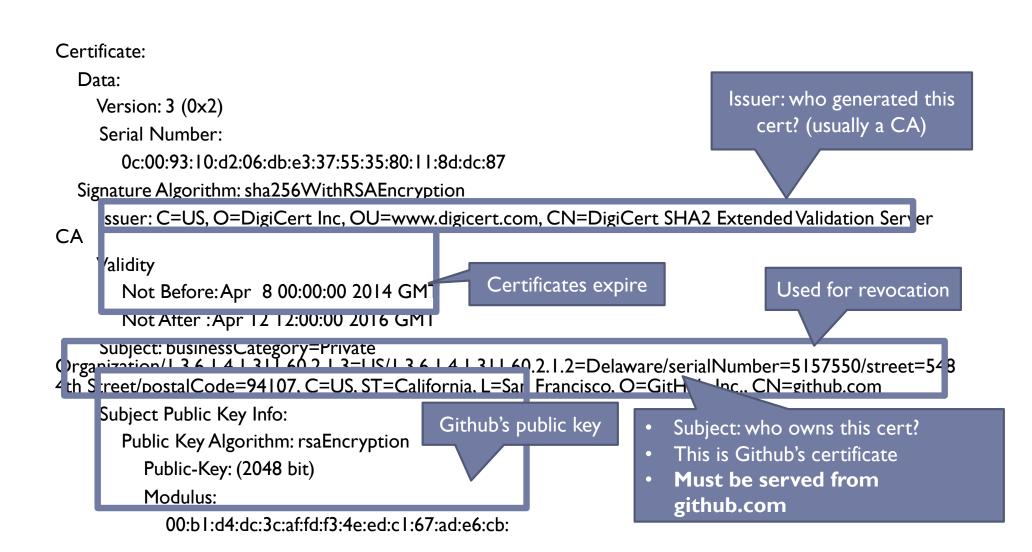
### Certificate Hierarchy - PKI



### Acquiring a Certificate



### X.509 Certificate



### Recover from secret key compromise

- Revocation is <u>very</u> important
- Many valid reasons to revoke a certificate
  - Private key corresponding to the certified public key has been compromised
  - User stopped paying his certification fee to the CA and the CA no longer wishes to certify him
  - CA's certificate has been compromised!

### Methods

- Certificate expiration
- Certificate revocation
  - Certificate Revocation Lists (CRL)
  - Online Certificate Status Protocol (OCSP)

### Expiration

- Certificate expiration is the simplest, most fundamental defense against secret key compromise
  - All certificates have an expiration date
  - A stolen key is only useful before it expires
- Ideally, all certs should have a short lifetime
  - Months, weeks, or even days
- Problem: most certs have multi-year lifetimes
  - This gives an attacker plenty of time to abuse a stolen key

#### X.509 Certificate

Validity

Not Before: Apr 8 00:00:00 2014 GMT Not After : Apr 12 12:00:00 2016 GMT

### Revocation

- Certificate revocations are another fundamental mechanism for mitigating secret key compromises
  - After a secret key has been compromised, the owner is supposed to revoke the certificate
- CA's are responsible for hosting databases of revoked certificates that they issued
- Clients are supposed to query the revocation status of all certificates they encounter during validation
  - If a certificate is revoked, the client should never accept it
- Two revocation protocols for TLS certificates
  - 1. Certificate Revocation Lists (CRLs): download list of revoked certificated
  - 2. Online Certificate Status Protocol (OCSP): API to query status of certificate

TLS

## What Is SSL / TLS?

- Secure Sockets Layer and Transport Layer Security protocols
  - Same protocol design, different crypto algorithms
- De facto standard for Internet security
  - "The primary goal of the TLS protocol is to provide privacy and data integrity between two communicating applications"
- Deployed in every Web browser; also VoIP, payment systems, distributed systems, etc

### SSL / TLS Guarantees

- End-to-end secure communications at transport layer in the presence of a network attacker
  - Attacker completely owns the network: controls Wi-Fi, DNS, routers, his own websites, can listen to any packet, modify packets in transit, inject his own packets into the network

#### Properties

- Authentication of server (optionally, client authentication)
- Confidentiality of communication
- Integrity against active attacks

### History of the Protocol

#### SSL I.0 – internal Netscape design, early 1994

- Lost in the mists of time
- SSL 2.0 Netscape, Nov 1994
  - Several weaknesses
- SSL 3.0 Netscape and Paul Kocher, Nov 1996
- TLS I.0 Internet standard, Jan 1999
  - Supersedes SSL: SSL is known to be insecure
  - Based on SSL 3.0, but not interoperable (uses different cryptographic algorithms)
- ▶ TLS I.I Apr 2006
- ▶ TLS I.2 Aug 2008
- ▶ TLS I.3 Aug. 2018

### **TLS Basics**

TLS consists of two protocols

### Handshake protocol

- Session initiation by client
- Uses public-key cryptography to establish several shared secret keys between the client and the server
- Server must have an asymmetric keypair
  - X.509 certificates contain signed public keys rooted in PKI

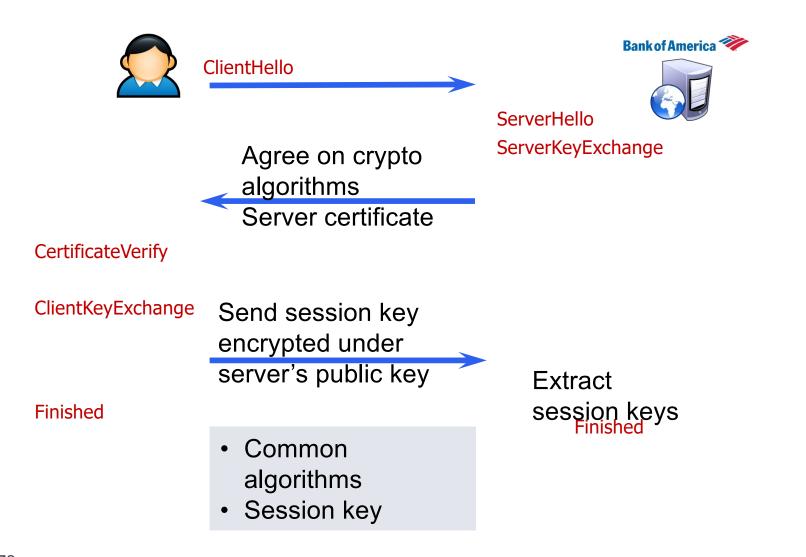
#### Record protocol

Uses the secret keys established in the handshake protocol to protect confidentiality and integrity of data exchange between the client and the server

### TLS Handshake Protocol

- Runs between a client and a server
  - Client = Web browser
  - Server = website
- Negotiate version of the protocol and the set of cryptographic algorithms to be used
  - Interoperability between different implementations
- Authenticate server
  - Use digital certificates to learn server's public keys and verify the certificate
  - Client authentication is optional
- Use public keys to establish a shared secret

### Handshake Protocol Structure



### Record Protocol Structure



#### Encryption (m)

- Mac-then-Enc using keys derived from the session key
- MAC uses a counter to prevent replay attacks
- Supports CBC-AES and HMAC-SHA1
- Provides authenticated encryption

Decryption(c)

First decrypt, then check MAC



#### Decryption(c)

First decrypt, then check MAC

#### Encryption (m)

Similar algorithm, but needs different keys (set of keys for each communication direction)